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How to Use This Addenda

Make sure you're ready to teach by noting the **Necessary Materials and Pre-Lesson Prep** you will need to gather or complete prior to the lesson

Find high-leverage instructional moves in the **Lesson Look Fors**. This is what leaders should see when observing your instruction

Note how your lesson objective ties to your state **Standards**

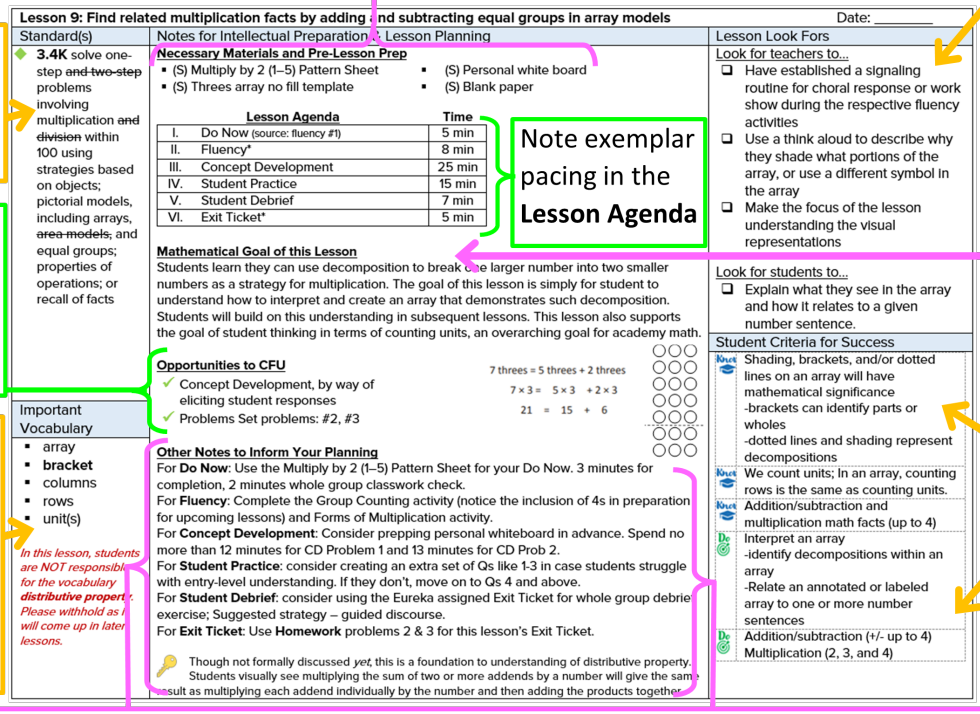
Note exemplar pacing in the **Lesson Agenda**

Use the **Mathematical Goal of the Lesson** to keep you focused on the appropriate student outcome

Plan purposeful questioning and responses using **Opportunities to CFU**

Plan to stress **Important Vocabulary** in the lesson. New vocab for the unit is indicated in bold

Plan instruction around what students need to Know & Do to be successful on the Exit Ticket using the identified **Student Know/Do Chart**



Lesson 9: Find related multiplication facts by adding and subtracting equal groups in array models Date: _____

Standard(s)

3.4K solve one-step ~~and two-step~~ problems involving multiplication ~~and division~~ within 100 using strategies based on objects; pictorial models, including arrays, ~~area models~~; and equal groups; properties of operations; or recall of facts

Important Vocabulary

- array
- **bracket**
- columns
- rows
- unit(s)

In this lesson, students are NOT responsible for the vocabulary ***distributive property****. Please withhold as it will come up in later lessons.*

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- (S) Multiply by 2 (1–5) Pattern Sheet
- (S) Threes array no fill template
- (S) Personal white board
- (S) Blank paper

Lesson Agenda		Time
I.	Do Now (source: fluency #1)	5 min
II.	Fluency*	8 min
III.	Concept Development	25 min
IV.	Student Practice	15 min
V.	Student Debrief	7 min
VI.	Exit Ticket*	5 min

Mathematical Goal of this Lesson

Students learn they can use decomposition to break one larger number into two smaller numbers as a strategy for multiplication. The goal of this lesson is simply for student to understand how to interpret and create an array that demonstrates such decomposition. Students will build on this understanding in subsequent lessons. This lesson also supports the goal of student thinking in terms of counting units, an overarching goal for academy math.

Opportunities to CFU

- ✓ Concept Development, by way of eliciting student responses
- ✓ Problems Set problems: #2, #3

7 threes = 5 threes + 2 threes
7 × 3 = 5 × 3 + 2 × 3
21 = 15 + 6

Other Notes to Inform Your Planning

For **Do Now**: Use the Multiply by 2 (1–5) Pattern Sheet for your Do Now. 3 minutes for completion, 2 minutes whole group classwork check.
For **Fluency**: Complete the Group Counting activity (notice the inclusion of 4s in preparation for upcoming lessons) and Forms of Multiplication activity.
For **Concept Development**: Consider prepping personal whiteboard in advance. Spend no more than 12 minutes for CD Problem 1 and 13 minutes for CD Prob 2.
For **Student Practice**: consider creating an extra set of Qs like 1-3 in case students struggle with entry-level understanding. If they don't, move on to Qs 4 and above.
For **Student Debrief**: consider using the Eureka assigned Exit Ticket for whole group debrie exercise; Suggested strategy – guided discourse.
For **Exit Ticket**: Use **Homework** problems 2 & 3 for this lesson's Exit Ticket.

Though not formally discussed *yet*, this is a foundation to understanding of distributive property. Students visually see multiplying the sum of two or more addends by a number will give the sam result as multiplying each addend individually by the number and then adding the products together.

Lesson Look Fors

Look for teachers to...

- ❑ Have established a signaling routine for choral response or work show during the respective fluency activities
- ❑ Use a think aloud to describe why they shade what portions of the array, or use a different symbol in the array
- ❑ Make the focus of the lesson understanding the visual representations

Look for students to...

- ❑ Explain what they see in the array and how it relates to a given number sentence.

Student Criteria for Success

Know: Shading, brackets, and/or dotted lines on an array will have mathematical significance
-brackets can identify parts or wholes
-dotted lines and shading represent decompositions

Know: We count units; In an array, counting rows is the same as counting units.

Know: Addition/subtraction and multiplication math facts (up to 4)

Do: Interpret an array
-identify decompositions within an array
-Relate an annotated or labeled array to one or more number sentences

Do: Addition/subtraction (+/- up to 4)
Multiplication (2, 3, and 4)

Find recommended lesson modifications, content knowledge boosters, and/or high-leverage instructional moves that may not be in your Teacher Edition located in **Other Notes to Inform Your Planning**

UNIT SYNOPSIS

Functions defined by polynomial expressions are called **polynomial functions**. The graphs of polynomial functions are *smooth* and *continuous* (further discussed in AP Calculus AB/BC) and can have many peaks and valleys, but one of the most notable characteristics of a polynomial is the relationship between the degree of a polynomial and the number of zeros it contains. To find these zeros, students will need to perform polynomial division techniques to solve polynomial equations.

If $P(x)$ and $Q(x)$ are polynomials functions with $Q(x) \neq 0$, then $P(x)/Q(x)$ is called a **rational function**. Unlike the graphs of polynomial functions, rational functions may contain holes (point discontinuities) and breaks (asymptotic discontinuities). To explore these discontinuities, students will need to divide and simplify polynomial functions in order to classify each type of discontinuity, if any. Students will be introduced to the limit notation when discussing end behavior of rational functions and calculating horizontal or oblique asymptotes. This unit extends the knowledge and skills of polynomial and rational functions to solve inequalities which is an important functional analysis skill for calculus courses.

This unit features topics in polynomial functions, rational functions, and solving inequalities.

CONTENT STANDARDS

Below are the standards addressed in this unit.

Texas Essential Knowledge and Skills (TEKS)	
Knowledge and Skills	**Student Expectations (SE)**
(2) Functions The student uses process standards in mathematics to **explore**, **describe**, and **analyze** the **attributes of functions**. The student makes connections between **multiple representations** of functions and **algebraically constructs** new functions. The student analyzes and uses functions to **model** real-world problems.	**(2.G)** Graph functions, including ~~exponential, logarithmic, sine, cosine~~, rational, polynomial, ~~and power functions~~ and their transformations, including $af(x)$, $f(x)+d$, $f(x-c)$, $f(bx)$ for specific values of a, b, c, and d, in mathematical and real-world problems. **(2.I)** Determine and analyze the key features of ~~exponential, logarithmic,~~ rational, polynomial, ~~power, trigonometric, inverse trigonometric, and piecewise defined functions~~, ~~including step functions~~ such as domain, range, symmetry, relative maximum/minimum, zeros, asymptotes, and intervals over which the function is increasing/decreasing. **(2.J)** Analyze and describe end behavior of functions, including ~~exponential, logarithmic,~~ rational, polynomial~~, and power functions,~~ using infinity notation to communicate this characteristic in mathematical and real-world problems. **(2.K)** Analyze characteristics of rational functions and the behavior of the function around the asymptotes, including horizontal, vertical, and oblique asymptotes. **(2.L)** Determine various types of discontinuities in the interval $(-\infty, \infty)$ as they relate to functions and explore the limitations of the graphing calculator as it relates to the behavior of the function around discontinuities.
(5) Algebraic Reasoning The student uses process standards in mathematics to **evaluate expressions**, **describe patterns**, **formulate models**, and **solve equations and inequalities** using properties, procedures, or algorithms.	**(5.J)** Solve polynomial equations with real coefficients by applying a variety of techniques in mathematical and real-world problems. **(5.K)** Solve polynomial inequalities with real coefficients by applying a variety of techniques and write the solution set of the polynomial inequality in interval notation in mathematical and real-world problems. **(5.L)** Solve rational inequalities with real coefficients by applying a variety of techniques and write the solution set of the rational inequality in interval notation in mathematical and real-world problems.

[2]Parts of standard that are crossed out are not taught in this unit but will be taught in future units.

Focus on Disciplinary Literacy 	Mathematical Process Standard (F) – Analyze mathematical relationships to connect and communicate mathematical ideas.
	Mathematical Process Standard (G) – Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

LEARNING SUPPORTS BY LESSON

There is a checkmark for the math support if the lesson	Lessons → Math Supports	L1	L2	L3	L4	L5	L6	L7	L8	L9
makes a connection to prior content or from a previous unit or academic year	Access Prior Knowledge	✓	✓	✓	✓	✓	✓	✓	✓	✓
uses familiar contexts or experiences to make the learning relevant to students	Real-World Connections						✓			
makes use of graphic organizers	Graphic Organizers	✓	✓				✓			✓
includes tools like rulers, protractors, patty paper, algebra tiles, etc.	Tools or Manipulatives									
incorporates tables, reference charts, displays, pictures, models, or color-coding	Visual Aids	✓	✓	✓	✓	✓	✓	✓	✓	✓
includes definitions, examples vs. nonexamples, cognates, etc.	Vocabulary Supports	✓	✓	✓	✓	✓	✓	✓	✓	✓
includes strategies that support language development										
asks students to discuss with their partner to prepare for whole class discussion	- Turn and Talk	✓	✓							✓
teacher facilitates a whole class discussion to debrief key learnings	- Guided Discussion	✓	✓	✓	✓	✓	✓	✓	✓	
asks students to think independently, test their idea with a partner, and share whole group	- Think, Pair, Share	✓	✓		✓		✓	✓	✓	✓
includes sentence stems to support students with explanations	- Sentence Stems		✓							
provides opportunities for students to work with a partner or a group	Peer Collaboration	✓	✓	✓	✓	✓	✓	✓	✓	✓
uses mnemonics such as SohCahToa	Mnemonics					✓			✓	
includes websites or equipment that enhances the lesson	Technological Support	✓	✓	✓	✓	✓	✓	✓	✓	
content can be presented in different forms										
uses hands-on tools or manipulatives to represent the math	- Concrete									
uses drawings to represent the math	- Pictorial	✓	✓	✓	✓	✓	✓	✓	✓	✓
uses numbers and number sentences to represent the math	- Abstract	✓	✓	✓	✓	✓	✓	✓	✓	✓

ROADMAP

AT A GLANCE: Unit 2 – Polynomial and Rational Functions			
Day	Date	Lesson	Lesson Title
1		1	Sketching Polynomials: Local Behavior
2		2	Sketching Polynomials: End Behavior
3		3	The Factor and Remainder Theorem
4		4	Rational Zeros Theorem
5			*Unit 2 Success Day Alpha – Review Polynomial Functions*
6		5	The Fundamental Theorem of Algebra
7		6	Solving Polynomial Inequalities and Applications
8			*Unit 2 Success Day Beta – Review FTA or Solving Polynomial Inequalities*
9		7	Graphing Rational Functions, Day 1
10		8	Graphing Rational Functions, Day 2
11		9	Solving Rational Inequalities and Applications
12			*Unit 2 Success Day Gamma – Review topics based on your data*
13			*Unit 2 Success Day Delta – Unit Assessment Review*
14			End of Unit 2 Assessment

Date: _____

Lesson 1: Sketching Polynomials: Local Behavior

Standard(s)

- **(2.G)** Graph functions, including ~~exponential, logarithmic, sine, cosine~~, rational, polynomial, ~~and power functions~~ and their transformations, including $af(x)$, $f(x)+d$, $f(x-c)$, $f(bx)$ for specific values of a, b, c, and d, in mathematical and real-world problems.
- **(2.I)** Determine and analyze the key features of ~~exponential, logarithmic,~~ rational, polynomial, ~~power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions~~ such as domain, range, symmetry, relative maximum/minimum, zeros, asymptotes, and intervals over which the function is increasing/decreasing.
- **(2.J)** Analyze and describe end behavior of functions, including ~~exponential, logarithmic,~~ rational, polynomial~~, and power functions,~~ using infinity notation to communicate this characteristic in mathematical and real-world problems.

Important Vocabulary

- **Polynomial**
- **Continuous**
- **Coefficients**
- **Leading term**
- **Extrema**
- **Turning points**
- **Multiplicity**
  - **Even/odd**
- **Repeated zero (root)**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Desmos (optional)
- Document camera
- Student laptops (optional)

Lesson Structure:

- Do Now (8 min)
- INM (27 min)
- Student Practice (13 min)
- Debrief (2 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will be able to sketch the graphs of polynomial function given in factored form and determine its local behavior.

Opportunities to CFU

- ✓ What can a polynomial function in factored form tell us about its local behavior? (INM)
- ✓ How we determine the degree of a polynomial function? How does that help us understand its local behavior? (INM)

Other Notes to Inform Your Planning

- By the end of the first quarter, students will be able to assess local behavior, end behavior and some aspects of continuity for polynomial and rational functions (Unit 2), exponential and logarithmic functions (Unit 3), and trigonometric functions (Unit 4).
- In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the **2008 AP Calculus AB released exam**.

22. If $f'(x) = (x-2)(x-3)^2(x-4)^3$, then f has which of the following relative extrema?

I. A relative maximum at $x = 2$

II. A relative minimum at $x = 3$

III. A relative maximum at $x = 4$

(A) I only

(B) III only

(C) I and III only

(D) II and III only

(E) I, II, and III

Lesson Look Fors

Look for teachers to…

- ❑ Guided discourse, probing questions, and pushing students' thinking.
- ❑ **Optional**: Uses the Desmos link to model problems from the handouts.

Look for students to…

- ❑ Articulate the degree of a polynomial function given in standard or factored form.
- ❑ Explain why the number of possible real zeros and turning points is n and $n-1$, respectively.
- ❑ Explain why some roots cross or bounce off the x-axis.

Student Know/Do Chart

Know	The degree of a polynomial can tell us the maximum number of x-intercepts and turning points.
Know	A polynomial in factored form will determine the number of real roots as well as their corresponding multiplicity.
Do	Graph polynomial functions given in factored form.
Do	State the degree, number of x-intercepts and their corresponding multiplicities given a polynomial function.

Date: _____

Lesson 2: Sketching Polynomials: End Behavior

Standard(s)

- **(2.G)** Graph functions, including ~~exponential, logarithmic, sine, cosine~~, rational, polynomial, ~~and power functions~~ and their transformations, including $af(x)$, $f(x)+d$, $f(x-c)$, $f(bx)$ for specific values of a, b, c, and d, in mathematical and real-world problems.
- **(2.I)** Determine and analyze the key features of ~~exponential, logarithmic,~~ rational, polynomial, ~~power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions~~ such as domain, range, symmetry, relative maximum/minimum, zeros, asymptotes, and intervals over which the function is increasing/decreasing.
- **(2.J)** Analyze and describe end behavior of functions, including ~~exponential, logarithmic,~~ rational, polynomial~~, and power functions,~~ using infinity notation to communicate this characteristic in mathematical and real-world problems.

Important Vocabulary

- **End Behavior**
- **Limit Notation**
- Polynomial
- Continuous
- Coefficients
- Leading term
- Extrema
- Turning points
- Multiplicity
  - Even/odd
- Repeated zero (root)

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Desmos (optional)
- Document camera
- Student laptops (optional)

Lesson Structure:

- Do Now (5 min)
- INM (30 min)
- Student Practice (15 min)
- Debrief (2 min)
- Exit Ticket (3 min)

Mathematical Goal of this Lesson

In this lesson, students will be able to sketch the graphs of polynomial function given in factored or standard form and determine its end behavior.

Opportunities to CFU

- ✓ What type of end behavior do polynomial functions of even degree have? (INM)
- ✓ What type of end behavior do polynomial functions of odd degree have? (INM)
- ✓ How does the leading coefficient affect end behavior?

Other Notes to Inform Your Planning

- By the end of the first quarter, students will be able to assess local behavior, end behavior and some aspects of continuity for polynomial and rational functions (Unit 2), exponential and logarithmic functions (Unit 3), and trigonometric functions (Unit 4).
- In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the 2012 AP Calculus AB released exam.

Focus on Disciplinary Literacy

INM & Debrief

2. Let f be the function given by $f(x) = 300x - x^3$. On which of the following intervals is the function f increasing?

(A) $(-\infty, -10]$ and $[10, \infty)$

(B) $[-10, 10]$

(C) $[0, 10]$ only

(D) $[0, 10\sqrt{3}]$ only

(E) $[0, \infty)$

Lesson Look Fors

Look for teachers to…

- ❑ Guided discourse, probing questions, and pushing students' thinking.
- ❑ **Optional**: Uses the Desmos link to model problems from the handouts.

Look for students to…

- ❑ Identifies and determines the degree of a polynomial function given in standard or factored form.
- ❑ Uses the degree of a polynomial function to explain its end behavior.

Student Know/Do Chart

Know	The degree of a polynomial function can determine the type of end behavior is has; same or opposite.
Know	The leading coefficient can affect the end behavior of a polynomial function.
Do	Determine end behavior of a polynomial function using limit notation given in standard or factored form.
Do	Describe how end behavior is affected by its leading coefficient.

Date: _____

Lesson 3: The Factor and Remainder Theorem

Standard(s)

- **(2.I)** Determine and analyze the key features of ~~exponential, logarithmic,~~ rational, polynomial, ~~power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions~~ such as domain, range, symmetry, relative maximum/minimum, zeros, asymptotes, and intervals over which the function is increasing/decreasing.
- **(2.J)** Analyze and describe end behavior of functions, including ~~exponential, logarithmic,~~ rational, polynomial~~, and power functions,~~ using infinity notation to communicate this characteristic in mathematical and real-world problems.

Important Vocabulary

- **Remainder**
- **"Divides evenly"**
- **Remainder Theorem**
- **Factor Theorem**
- **Irreducible Factor**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Desmos (optional)
- Document camera
- Student laptops (optional)

Lesson Structure:

- Do Now (10 min)
- INM (20 min)
- Student Practice (15 min)
- Debrief (3 min)
- Exit Ticket (7 min)

Mathematical Goal of this Lesson

Today's lesson is a deeper examination into local behavior of polynomial functions with an emphasis on zeros and how they are connected to factors and remainders using long division. *It is suggested to review long and synthetic division before this lesson.*

Opportunities to CFU

- ✓ How are factors of a polynomial function connected to its roots for its graph? (Beginning of INM)
- ✓ How can we verify $x = c$ is a zero?
- ✓ How can we determine if a factor divides a polynomial function evenly? (End of INM)

Other Notes to Inform Your Planning

- By the end of the first quarter, students will be able to assess local behavior, end behavior and some aspects of continuity for polynomial and rational functions (Unit 2), exponential and logarithmic functions (Unit 3), and trigonometric functions (Unit 4).
- In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the **2012 AP Calculus AB released exam**.

2. Let f be the function given by $f(x) = 300x - x^3$. On which of the following intervals is the function f increasing?

(A) $(-\infty, -10]$ and $[10, \infty)$

(B) $[-10, 10]$

(C) $[0, 10]$ only

(D) $[0, 10\sqrt{3}]$ only

(E) $[0, \infty)$

Lesson Look Fors

Look for teachers to…

- ❑ Guided discourse, probing questions, and pushing students' thinking.
- ❑ **Optional**: Uses the Desmos link to model problems from the handouts.

Look for students to…

- ❑ Articulate the Factor Theorem to assess whether:
  - $x - c$ is a factor of a polynomial function f.
  - $x = c$ is a zero of of a polynomial function f.

Student Know/Do Chart

Know: If $f(c) = 0$, then $x = c$ is a solution or root of the polynomial equation $f(x) = 0$.

Know: If $f(c) = 0$, then $x = c$ is a solution or root of the polynomial equation $f(x) = 0$.

Do: Apply the Factor Theorem to verify that $x = c$ is a solution to $f(x) = 0$.

Do: Apply the Factor Theorem to verify that $x = c$ is a solution to $f(x) = 0$.

Do: Given a factor of a polynomial function f, determine the remaining zeros of f.

Date: _____

Lesson 4: Rational Root Theorem

Standard(s)

- **(5.J)** Solve polynomial equations with real coefficients by applying a variety of techniques in mathematical and real-world problems.

Important Vocabulary

- **Rational zeros (roots)**
- **Irrational zeros**
- **Rational Root Theorem**
- **Descartes' Rule of Signs**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Document camera

Lesson Structure:

- Do Now (10 min)
- INM (23 min)
- Student Practice (14 min)
- Debrief (2 min)
- Exit Ticket (6 min)

Mathematical Goal of this Lesson

In this lesson, students will have a deeper understanding of the real zeros of polynomial functions. Students are able to generate a list of possible rational zeros and use them to find the actual real zeros as well as infer the number of real zeros via Descartes' Rule of Signs. This lesson will have an example where complex zeros are involved but will be formalized in Lesson 5.

Opportunities to CFU

- ✓ How can we factor a polynomial function with at least degree of 3? (INM)
- ✓ How do the number of sign changes of the terms of a polynomial function f help infer about the real zeros of f ? (Debrief)

Other Notes to Inform Your Planning

- By the end of the first quarter, students will be able to assess local behavior, end behavior and some aspects of continuity for polynomial and rational functions (Unit 2), exponential and logarithmic functions (Unit 3), and trigonometric functions (Unit 4).
- In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the **2008 AP Calculus AB released exam**.

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5. The function f given by $f(x) = 2x^3 - 3x^2 - 12x$ has a relative minimum at $x =$

(A) -1 (B) 0 (C) 2 (D) $\frac{3-\sqrt{105}}{4}$ (E) $\frac{3+\sqrt{105}}{4}$

Lesson Look Fors

Look for teachers to…

- ❑ Guided discourse, probing questions, and pushing students' thinking.

Look for students to…

- ❑ Articulate the RTT.
- ❑ Apply the Factor Theorem or polynomial division to verify which possible rational zeros are indeed real zeros of a polynomial function.
- ❑ Articulate Descartes' Rule of Signs.

Student Know/Do Chart

Know	The RRT generates a list of all possible rational zeros for a polynomial function f.
Know	Descartes' Rule of Sign can help us infer on the number of possible, positive or negative, real zeros of f.
Do	Create a list of possible rational zeros.
Do	Use the Factor Theorem or long division to verify real zeros of a polynomial function f. Repeat, as needed.
Do	Determine the number of possible positive and negative real zeros of a polynomial function f.

Date: _____

Lesson 5: The Fundamental Theorem of Algebra

Standard(s)

- **(5.J)** Solve polynomial equations with real coefficients by applying a variety of techniques in mathematical and real-world problems.

Important Vocabulary

- **Fundamental Theorem of Algebra**
- **Linear Factorization Theorem**
- **Complex Conjugate**
- **Conjugate Pairs**
- **"Irreducible over the reals"**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Document camera

Lesson Structure:

- Do Now (8 min)
- INM (25 min)
- Student Practice (15 min)
- Debrief (2 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this final lesson on polynomial functions, we extend the field of real zeros of a polynomial function to include the complex numbers, $a + bi$, where $a, b \in \mathbb{R}$. The process in finding complex zeros is the same as the previous lessons. Some examples may include quadratic factor that are irreducible over $\mathbb{R}$.

Opportunities to CFU

- ✓ What do polynomial functions containing an irreducible factor indicate? (INM)
- ✓ How is the FTA different from what was discussed in Lesson 1 about the zeros of a polynomial?

Other Notes to Inform Your Planning

- By the end of the first quarter, students will be able to assess local behavior, end behavior and some aspects of continuity for polynomial and rational functions (Unit 2), exponential and logarithmic functions (Unit 3), and trigonometric functions (Unit 4).
- In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the **2012 AP Calculus AB released exam**.

2. Let f be the function given by $f(x) = 300x - x^3$. On which of the following intervals is the function f increasing?

(A) $(-\infty, -10]$ and $[10, \infty)$

(B) $[-10, 10]$

(C) $[0, 10]$ only

(D) $[0, 10\sqrt{3}]$ only

(E) $[0, \infty)$

Lesson Look Fors

Look for teachers to…

- ❑ Guided discourse, probing questions, and pushing students' thinking.
- ❑ **Optional**: Uses the Desmos link to model the application problem in the Do Now.

Look for students to…

- ❑ Unpack conditional statements, e.g., IF-THEN statements by identifying
  - Conditions
  - Conclusion
- ❑ Prior lesson connections in their work, such as
  - Rational root theorem
  - Factor Theorem
  - Long or synthetic division
  - Describing polynomial local/end behavior

Student Know/Do Chart

Know: The FTA states that any polynomial with degree n and real coefficients has *exactly* n complex zeros.

Know: All polynomials of degree n with real coefficient can be expressed as a product of linear factors over $\mathbb{C}$.

Do: Apply the conjugate theorem.

Do: Determine ALL zeros of a polynomial function.

Do: Write a polynomial function as a product of linear factors over $\mathbb{C}$.

Date: _____

Lesson 6: Solving Polynomial Inequalities and Applications

Standard(s)

- **(5.K)** Solve polynomial inequalities with real coefficients by applying a variety of techniques and write the solution set of the polynomial inequality in interval notation in mathematical and real-world problems.

Important Vocabulary

- **Polynomial Inequality**
- **Solution set**
- **Sign chart(s)**
- Interval notation

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- GeoGebra Activity (optional)
- Desmos Activity (optional)
- Document camera
- Student laptops (optional)

Lesson Structure:

- Do Now (15 min)
- INM (15 min)
- Student Practice (18 min)
- Debrief (2 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

This lesson encompasses the knowledge and skills about polynomial functions as it extends to polynomial equalities. Students will be able to solve polynomial inequalities in abstract and contextual settings.

Opportunities to CFU

- ✓ How is solving polynomial equations different from solving polynomial inequalities?
- ✓ How does a solution set change in an inequality from $\leq$ to $<$ (or from $\geq$ to $>$)?

Other Notes to Inform Your Planning

- Solving inequalities will be revisited in Unit 2, Lesson 9.
- In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the 2012 AP Calculus AB released exam.

2. Let f be the function given by $f(x) = 300x - x^3$. On which of the following intervals is the function f increasing?

(A) $(-\infty, -10]$ and $[10, \infty)$

(B) $[-10, 10]$

(C) $[0, 10]$ only

(D) $[0, 10\sqrt{3}]$ only

(E) $[0, \infty)$

Lesson Look Fors

Look for teachers to…

- ❑ Guided discourse, probing questions, and pushing students' thinking.

Look for students to…

- ❑ Utilize a sign chart or a graph to find the values of x that satisfy the inequality.
- ❑ Give rationale as why their solution set satisfies the inequality.

Student Know/Do Chart

Know — The solution for a polynomial inequality is an interval. End points of intervals may be open and or closed.

Know — When a function value is < 0, the graph is below the x-axis. When a function value is >0, the graph is above the x-axis.

Do — Utilize a sign chart or a graph to find the values of x that satisfy the inequality.

Do — Write solution sets in interval notation.

Date: _____

Lesson 7: Graphing Rational Functions, Day 1

Standard(s)

- **(2.I)** Determine and analyze the key features of ~~exponential, logarithmic,~~ rational, polynomial, ~~power, trigonometric, inverse trigonometric, and piecewise-defined functions, including step functions~~ such as domain, range, symmetry, relative maximum/minimum, zeros, asymptotes, and intervals over which the function is increasing/decreasing.
- **(2.K)** Analyze characteristics of rational functions and the behavior of the function around the asymptotes, including horizontal, vertical, and oblique asymptotes.
- **(2.L)** Determine various types of discontinuities in the interval $(-\infty, \infty)$ as they relate to functions and explore the limitations of the graphing calculator as it relates to the behavior of the function around discontinuities.

Important Vocabulary

- **Rational functions**
- **Vertical asymptotes**
- **Removable discontinuity**
- Domain
- Range
- x-intercept

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Desmos Card Match
- Posters/Whiteboards
- Colored pencils/Dry erase markers

Lesson Structure:

- Do Now (15 min)
- INM (15 min)
- Student Practice (15 min)
- Debrief (3 min)
- Exit Ticket (7 min)

Mathematical Goal of this Lesson

This lesson is the first in a two-lesson series on graphing rational functions. Students will be able to explore rational functions by investigating domain, intercepts, horizontal and single vertical asymptotes, and removable discontinuities.

Opportunities to CFU

- ✓ How do we determine infinite discontinuities? (Do Now)
- ✓ When do removable discontinuities occur? (INM)

Other Notes to Inform Your Planning

- By the end of the first quarter, students will be able to assess local behavior, end behavior and some aspects of continuity for polynomial and rational functions (Unit 2), exponential and logarithmic functions (Unit 3), and trigonometric functions (Unit 4).
- In AP Calculus, students learn to differentiate rational functions using the quotient rule.
- In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the 2008 AP Calculus AB released exam.

3. For $x \geq 0$, the horizontal line $y = 2$ is an asymptote for the graph of the function f. Which of the following statements must be true?

(A) $f(0) = 2$

(B) $f(x) \neq 2$ for all $x \geq 0$

(C) $f(2)$ is undefined.

(D) $\lim_{x \to 2} f(x) = \infty$

(E) $\lim_{x \to \infty} f(x) = 2$

Lesson Look Fors

Look for teachers to…

- ❑ Guided discourse, probing questions, and pushing students' thinking.
- ❑ Engage students in a card matching activity during Student Practice (Desmos is optional).

Look for students to…

- ❑ Explain when rational functions have infinite and removable discontinuities.
- ❑ Explain why rational functions may contain gaps in their domain.

Student Know/Do Chart

Know — Rational functions are in the form of $f(x)/g(x)$ where f and g are both polynomial functions and $g(x)$ is at least degree 1.

Know — Rational functions may contain infinite and removable discontinuities.

Do — Completely factor and simplify rational functions to determine discontinuities.

Do — Determine intercepts and domain of rational functions.

Date: _____

Lesson 8: Graphing Rational Functions: End Behavior

Standard(s)

- **(2.I)** Determine and analyze the key features of ~~exponential, logarithmic,~~ rational, polynomial, ~~power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions~~ such as domain, range, symmetry, relative maximum/minimum, zeros, asymptotes, and intervals over which the function is increasing/decreasing.
- **(2.K)** Analyze characteristics of rational functions and the behavior of the function around the asymptotes, including horizontal, vertical, and oblique asymptotes.
- **(2.L)** Determine various types of discontinuities in the interval $(-\infty, \infty)$ as they relate to functions and explore the limitations of the graphing calculator as it relates to the behavior of the function around discontinuities.

Important Vocabulary

- **End behavior asymptotes**
- Rational functions
- Vertical asymptotes
- Removable discontinuity
- Domain
- Range
- x-intercept

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Posters/Whiteboards
- Colored pencils/Dry erase markers

Lesson Structure:

- Do Now (8 min)
- INM (22 min)
- Student Practice (20 min)
- Debrief (2 min)
- Exit Ticket (3 min)

Mathematical Goal of this Lesson

This lesson is the second in a two-lesson series on graphing rational functions. Students will extend on the previous lesson by exploring and determining end behavior asymptotes; horizontal and oblique.

Opportunities to CFU

- ✓ When do rational functions have horizontal asymptotes? (INM)
- ✓ When do rational functions have oblique asymptotes? (INM)

Other Notes to Inform Your Planning

- By the end of the first quarter, students will be able to assess local behavior, end behavior and some aspects of continuity for polynomial and rational functions (Unit 2), exponential and logarithmic functions (Unit 3), and trigonometric functions (Unit 4).
- In AP Calculus, students learn to differentiate rational functions using the quotient rule.
- In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the 2008 AP Calculus AB released exam.

3. For $x \geq 0$, the horizontal line $y = 2$ is an asymptote for the graph of the function f. Which of the following statements must be true?

(A) $f(0) = 2$

(B) $f(x) \neq 2$ for all $x \geq 0$

(C) $f(2)$ is undefined.

(D) $\lim_{x \to 2} f(x) = \infty$

(E) $\lim_{x \to \infty} f(x) = 2$

Lesson Look Fors

Look for teachers to…

- ❑ Guided discourse, probing questions, and pushing students' thinking.

Look for students to…

- ❑ Explain when rational functions have infinite and removable discontinuities.
- ❑ Explain why rational functions may contain gaps in their domain.

Student Know/Do Chart

Know — Rational functions are in the form of $f(x)/g(x)$ where f and g are both polynomial functions and $g(x)$ is at least degree 1.

Know — Rational functions may contain infinite and or removable discontinuities.

Do — Completely factor and simplify rational functions to determine all asymptotes.

Do — Determine intercepts and domain of rational functions.

Date: _____

Lesson 9: Rational Inequalities and Applications

Standard(s)

- **(5.L)** Solve rational inequalities with real coefficients by applying a variety of techniques and write the solution set of the rational inequality in interval notation in mathematical and real-world problems.

Important Vocabulary

- **Rational inequality**
- Solution set
- Sign chart
- Interval notation

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Dry erase markers/colored pencils
- Whiteboards/posters

Lesson Structure:

- Do Now (15 min)
- INM (15 min)
- Student Practice (17 min)
- Debrief (2 min)
- Exit Ticket (6 min)

Mathematical Goal of this Lesson

In this lesson, students will use their previous knowledge and skills from rational functions to solve rational inequalities using a sign chart and verifying their solutions graphically.

Opportunities to CFU

- ✓ How is solving polynomial equations different from solving polynomial inequalities?
- ✓ How does a solution set change in an inequality from $\leq$ to $<$ (or from $\geq$ to $>$)?

Other Notes to Inform Your Planning

- By the end of the first quarter, students will be able to assess local behavior, end behavior and some aspects of continuity for polynomial and rational functions (Unit 2), exponential and logarithmic functions (Unit 3), and trigonometric functions (Unit 4).
- In AP Calculus, students learn to differentiate rational functions using the quotient rule.
- In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the 2003 AP Calculus AB released exam.

15. Let f be the function with derivative given by $f'(x) = x^2 - \frac{2}{x}$. On which of the following intervals is f decreasing?

(A) $(-\infty, -1]$ only

(B) $(-\infty, 0)$

(C) $[-1, 0)$ only

(D) $\left(0, \sqrt[3]{2}\right]$

(E) $\left[\sqrt[3]{2}, \infty\right)$

Lesson Look Fors

Look for teachers to…

- ❑ Guided discourse, probing questions, and pushing students' thinking.

Look for students to…

- ❑ Utilize a sign chart or a graph to find the values of x that satisfy the inequality.
- ❑ Give rationale as why their solution set satisfies the inequality.

Student Know/Do Chart

Know — The solution for a rational inequality is an interval. End points of intervals may be open and or closed.

Know — When a function value is < 0, the graph is below the x-axis. When a function value is >0, the graph is above the x-axis.

Do — Utilize a sign chart or a graph to find the values of x that satisfy the inequality.

Do — Write solution sets in interval notation.

Recommended Unit 2 Success Day Material and Resources

Date: _____

To review topics based on your data on Success Days, use the following resources. Your exit ticket data should be used to determine individualized needs. The resources can be used in small groups, whole groups, or independent groups and be integrated with other classroom routines, like computer aligned practice and teacher-led groups.

To review or practice Sketching Polynomials:

Instructional Resources:

- Factoring Quadratic Expressions Practice
- Polynomial Functions PPT
- Polynomials and End Behavior PPT
- Leading Coefficient Test Exploration Activity

Content Video Lessons:

- Introduction to Polynomial Functions
- Introduction to Polynomials
- Polynomial End Behavior
- Graphing Polynomial Functions
- Finding Zeros and Their Multiplicities for a Polynomial
- End Behavior of Graphs of Polynomial Functions

To review or practice Polynomial Division:

Instructional Resources:

- Long Division Guided Notes
- Polynomial Division PPT

Content Video Lessons:

- Using Long Division with Polynomials
- Using Synthetic Division with Polynomials
- The Remainder Theorem
- The Factor Theorem

To review or practice Rational Zeros Theorem:

Instructional Resources:

- Rational Zeros Guided Practice
- Rational Zeros Theorem PPT

Content Video Lessons:

- The Rational Zero Theorem
- Finding All Zeros of a Polynomial Equation
- Finding All Zeros of a Polynomial Equation: Another Example
- Descartes' Rule of Signs

To review or practice Fundamental Theorem of Algebra:

Instructional Resources:

- Fundamental Theorem of Algebra Practice (answers)
- Khan Academy Fundamental Theorem of Algebra

Content Video Lessons:

- The Fundamental Theorem of Algebra
- The Conjugate Pair Theorem

To review or practice Graphing Rational Functions:

Instructional Resources:

- Rational Functions PPT
- Khan Academy Rational Functions

Content Video Lessons:

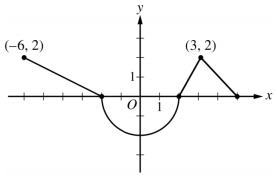
- Graphing Basic Rational Functions
- Finding the Vertical Asymptotes of a Rational Function
- Graphing Rational Functions with Vertical Asymptotes
- Graphing Rational Functions with Vertical and Horizontal Asymptotes

UNPACKED STANDARDS

Focus standards for this unit.

Standards Clarification

Standards	Specificity	Notes/Explanations/Examples

(2.I) ~~Determine~~ and analyze the key features of exponential, logarithmic, rational, polynomial, power, trigonometric, ~~inverse trigonometric~~, and piecewise defined functions, including step functions such as domain, range, symmetry, relative maximum/minimum, zeros, asymptotes, and intervals over which the function is increasing/decreasing.	**Concepts:**
	Parent Functions:
	• Exponential: $f(x) = a \cdot b^x$
	• Logarithmic: $f(x) = \log_b(x)$
	• Rational: $f(x) = \frac{1}{x}$
	• Polynomial: $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$
	• Power: $f(x) = a \cdot x^b$
	• Trigonometric: $f(x) = \sin(x)$ or $f(x) = \cos(x)$
	• Inverse Trigonometric $f(x) = \arcsin(x)$ or $f(x) = \arccos(x)$
	• Piecewise-defined functions
	• Key features
	  ○ Domain (interval notation*)
	  ○ Range (interval notation*)
	  ○ Symmetry (about x, y, origin)
	  ○ Maximum (local vs. absolute*)
	  ○ Minimum (local vs. absolute*)
	  ○ Extrema*
	  ○ Boundedness*
	  ○ Zeros
	  ○ Asymptotes
	  ○ Intervals of increasing (interval notation*)
	  ○ Intervals of decreasing (interval notation*)
	Content:
	Including, but not limited to:
	• Identify the parent function
	• Graph the function
	• Determine key features of the function
	• Analyze the function by the key features
		Below are the content and skill connections between the Algebra 1 TEKS and the AP standard.
		Algebra 2 TEKS
		(2A.2.A) graph the functions $f(x) = \sqrt{x}$, $f(x) = \frac{1}{x}$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = \lvert x \rvert$, and $f(x) = \log_b x$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum & minimum given an interval
		AP Calculus AB 2017 Released FRQ #3 Parts b & c
		 
		
		Graph of f'
		3. The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f', the derivative of f, consists of a semicircle and three line segments, as shown in the figure above.
		(a) Find the values of $f(-6)$ and $f(5)$.
		(b) On what intervals is f increasing? Justify your answer.
		(c) Find the absolute minimum value of f on the closed interval $[-6, 5]$. Justify your answer.
		(b) $f'(x) > 0$ on the intervals $[-6, -2)$ and $(2, 5)$. Therefore, f is increasing on the intervals $[-6, -2]$ and $[2, 5]$. — 2 : answer with justification
		(c) The absolute minimum will occur at a critical point where $f'(x) = 0$ or at an endpoint. — 2 : {1 : considers $x = 2$; 1 : answer with justification}
		$f'(x) = 0 \Rightarrow x = -2, x = 2$
		x | $f(x)$
		-6 | 3
		-2 | 7
		2 | $7 - 2\pi$
		5 | $10 - 2\pi$
		The absolute minimum value is $f(2) = 7 - 2\pi$.

VERTICAL STANDARDS

This section details the **progression** of key student expectations/standards** in the courses **before** and **after** this course. This will help you understand what **prior knowledge skills to build upon** and guide you in knowing what **skills you are preparing your students** for in the subsequent course.

Algebra 2	Pre-Calculus	AP Calculus AB Essential Knowledge
• **2A.2A** Graph the functions $f(x) = \sqrt{x}$, $f(x) = \frac{1}{x}$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = \lvert x \rvert$, and $f(x) = \log_b x$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum & minimum given an interval. • **2A.2B** Graph and write the inverse of a function using notation such as $f^{-1}(x)$. • **2A.2C** Describe and analyze the relationship between a function and its inverse (quadratic and square root, logarithmic and exponential), including the restriction(s) on domain, which will restrict its range. • **2A.2D** Use the composition of two functions, including the necessary restrictions on the domain, to determine if the functions are inverses of each other. • When $f(x)$ is replaced including $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific positive and negative values of a, b, c, and d, determine the effect on the: ○ **2A.4C** Graph of $f(x) = \sqrt{x}$ ○ **2A.5A** Graph of $f(x) = b^x$ and $f(x) = \log_b x$ where b is 2, 10, and e ○ **2A.6A** Graph of $f(x) = x^3$ and $f(x) = \sqrt[3]{x}$ ○ **2A.6C** Graph of $f(x) = \lvert x \rvert$ ○ **2A.6G** Graph of $f(x) = \frac{1}{x}$	• **(2.A)** Use the composition of two functions to model and solve real-world problems. • **(2.B)** Demonstrate that function composition is not always commutative. • **(2.C)** Represent a given function as a composite function of two or more functions. • **(2.D)** Describe symmetry of graphs of even and odd functions. • **(2.E)** Determine an inverse function, when it exists, for a given function over its domain or a subset of its domain and represent the inverse using multiple representations. • **(2.F)** Graph exponential, logarithmic, rational, polynomial, power, trigonometric, ~~inverse trigonometric~~, and piecewise defined functions, including step functions. • **(2.G)** Graph functions, including exponential, logarithmic, ~~sine, cosine~~, rational, polynomial, and power functions and their transformations, including $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d, in mathematical and real-world problems. • **(2.I)** Determine and analyze the key features of exponential, logarithmic, rational, polynomial, power, trigonometric, ~~inverse trigonometric~~, and piecewise defined functions, including step functions such as domain, range, symmetry, relative maximum, relative minimum, zeros, asymptotes, and intervals over which the function is increasing or decreasing.	• **(LIM-1.D.1)** Limits of sums, differences, products, quotients, and composite functions can be found using limit theorems. • **(LIM-2.B.2)** Polynomial, rational, power, exponential, logarithmic, and trigonometric functions are continuous at all points in their domains. • **(FUN-4.A.2)** The first derivative of a function can determine the location of relative (local) extrema of the function. • **(FUN-4.A.3)** Absolute (global) extrema of a function on a closed interval can only occur at critical points or at endpoints. • **(FUN-4.A.7)** The second derivative of a function may determine whether a critical point is the location of a relative (local) maximum or minimum. • **(FUN-4.A.8)** When a continuous function has only one critical point on an interval on its domain and the critical point corresponds to a relative (local) extremum of the function on the interval, then that critical point also corresponds to the absolute (global) extremum of the function on the interval. • **(FUN-7.E.3)** Solutions to differential equations may be subject to domain restrictions. NOTE: These “essential knowledge” (EK) standards are from The College Board Course Exam Description for AP Calculus AB/BC, not the TEKS.